Wormholes in spacetimes with cosmological horizons

Carlos Barceló

Instituto de Astrofísica de Andalucía, CSIC
Camino Bajo de Huétor, 18080 Granada, Spain

and

Instituto de Matemáticas y Física Fundamental, CSIC
C/ Serrano 121, 28006 Madrid, Spain

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Abstract

A generalisation of the asymptotic wormhole boundary condition for the case of spacetimes with a cosmological horizon is proposed. In particular, we consider de Sitter spacetime with small cosmological constant. The wave functions selected by this proposal are exponentially damped in WKB approximation when the scale factor is large but still much smaller than the horizon size. In addition, they only include outgoing gravitational modes in the region beyond the horizon. We argue that these wave functions represent quantum wormholes and compute the local effective interactions induced by them in low-energy field theory. These effective interactions differ from those for flat spacetime in terms that explicitly depend on the cosmological constant.

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1 Introduction

Wormholes are spacetime fluctuations that involve baby universes branching off and joining onto different regions of spacetime. In the dilute wormhole approximation, each wormhole end is considered to be connected to a different asymptotically large region of spacetime [1, 2]. In this situation, wormholes can be represented quantum mechanically by wave functions that satisfy the Wheeler-De Witt (WDW) equation. In order to recover the semiclassical behaviour expected for a wormhole, Hawking and Page [3] proposed that the wormhole wave functions should be exponentially damped for large three-geometries. Besides, these wave functions should be regular when the three-geometry degenerates to zero [3]. These boundary conditions are usually employed to select the wormhole wave functions among the solutions to the WDW equation.

It has been claimed that the existence of wormhole insertions in spacetime introduces local effective interactions in low-energy field theory and may modify the constants of nature [1, 4]. The analysis of the wormhole effects in spacetimes with cosmological horizons is particularly relevant. Spacetimes that describe solutions of interest in cosmology usually posses this kind of horizons. In addition, these horizons are generally present when the cosmological constant, Λ, is positive. Actually, the existence of quantum wormholes in spacetimes with positive Λ was already assumed by Coleman when putting forward his mechanism for the vanishing of the observed cosmological constant [5]. However, when

one tries to study wormhole processes in spacetimes with a cosmological horizon, one soon realises that the asymptotic boundary condition, as proposed by Hawking and Page, is no longer applicable. All the solutions to the WDW equation turn out to exhibit an oscillatory behaviour when the scale factor of the three-geometry becomes greater than the horizon size. In fact, the presence of a cosmological horizon in a Lorentzian spacetime implies that its Euclidean counterpart is compact in Euclidean time and, as a consequence, there does not exist an asymptotically large Euclidean region around the wormhole end.

In this work, we will propose a generalisation of the asymptotic wormhole boundary condition for spacetimes with a large cosmological horizon. According to this proposal, quantum wormholes can be represented by wave functions that have an exponentially damped WKB behaviour in the region of large three-geometries well inside the horizon and include only outgoing gravitational modes when the three-geometry is asymptotically large. Note, on the other hand, that the condition that the wormhole wave functions are regular when the three-geometry degenerates needs in principle no modification, because the presence of a cosmological horizon affects only the large scale behaviour.

We will particularise our discussion to the simplest gravitational system that presents a cosmological horizon, namely de Sitter spacetime. Together with the asymptotically flat and anti-de Sitter cases (which have already been considered in the literature [6, 7]), our analysis exhausts the study of wormholes in maximally symmetric spacetimes. We will see that our generalised wormhole boundary condition requires the wormhole throat to be much smaller than the existing horizon. Otherwise, the baby universe fluctuation could not be distinguished from the background spacetime. In this sense, we will understand that the expression "quantum wormhole" refers only to tunnelling processes that occur in regions well inside the cosmological horizon. In addition, we will assume that the cosmological horizon is large.

The local effective interactions produced by wormholes have been explicitly computed for asymptotically flat [6] and asymptotically anti-de Sitter wormholes [7] with a variety of matter field contents. Using our generalised wormhole boundary condition, we will calculate the effective interactions induced by wormholes in de Sitter spacetime. We will show that the existence of a cosmological horizon modifies these interactions with respect to the flat case by introducing terms that are proportional to even powers of the inverse horizon size.

In section 2, we generalise the asymptotic wormhole boundary condition and obtain the de Sitter wormhole wave functions. Sec. 3 deals with the effective interactions induced by these wormholes in low-energy field theory. In both sections, we work with a conformal scalar field as the matter content. Finally, we discuss our results and their generalisation to other matter fields in Sec. 4.

2   De Sitter wave functions

Let us analyse quantum mechanically a gravitational system with positive cosmological constant and a conformally coupled scalar field. In the following, a and χ_1 denote the scale factor of the sections of constant time and the homogeneous mode of the conformal scalar field on these sections, respectively. These configuration variables will be treated exactly. We will also consider the deviations from the homogeneous and isotropic configuration

described by a and χ_1, but only up to first order of perturbation theory. These deviations will be described by the coefficients of the expansion of the conformal field in hyperspherical harmonics on the three-sphere (in this process, gravitational waves are neglected and the gravitational harmonics are gauged away [7, 8]). Explicitly, we decompose the scalar field as[1]

$$\phi = \sqrt{\frac{1}{2\pi^2}} a^{-1} \sum_{n,\sigma_n} \chi_{n\sigma_n} Q^{n\sigma_n}, \tag{2.1}$$

where $Q^{n\sigma_n}$ are the scalar harmonics, eigenfunctions of the Laplace-Beltrami operator in the three-sphere with eigenvalues $-(n^2 - 1)$, the index σ_n runs over a basis of the corresponding degenerate eigenspace [7, 8], and the coefficients $\chi_{n\sigma_n}$ depend only on the time coordinate. In conformal time, the action for the system can then be written

$$
\begin{aligned}
I &= \int d\eta (\pi_a a' + \pi_n \chi'_n - NH), \\
H &= \frac{1}{2}\left(-\pi_a^2 + a^2 - \lambda a^4 + \sum_n (\pi_n^2 - n^2 \chi_n^2) \right).
\end{aligned} \tag{2.2}
$$

Here, π_a and π_n are the momenta canonically conjugate to a and χ_n, N is the lapse function, and the prime denotes derivative with respect to η. From Eq. (2.2), we obtain the WDW equation

$$\left[-\frac{\partial^2}{\partial a^2} + a^2 - \lambda a^4 - \sum_{n,\sigma_n}\left(-\frac{\partial^2}{\partial \chi_{n\sigma_n}^2} + n^2 \chi_{n\sigma_n}^2 - \frac{1}{2} \right) \right] \Psi(a, \chi_{n\sigma_n}) = 0, \tag{2.3}$$

in which we have chosen an operator ordering that removes the ground-state energy of each of the harmonic oscillators. We can solve this equation by separation of variables. By imposing standard boundary conditions for the quantum harmonic oscillators, and restricting all considerations to rotationally invariant states of the scalar field [7, 9] (i.e., states that depend on the inhomogeneous configuration variables only through the rotationally invariant combinations $\chi_n^2 = \sum_{\sigma_n} \chi_{n\sigma_n}^2$), we arrive at wave functions of the form

$$\Psi_{N_1,\cdots,N_n,\cdots}(a, \chi_n) = \psi_E(a) \mathcal{H}_{N_1}(\chi_1) e^{-\frac{1}{2}\chi_1^2} \prod_{n>1} \mathcal{L}_{N_n}^{(n^2-3)/2}(n\chi_n^2) e^{-\frac{1}{2}n\chi_n^2}, \tag{2.4}$$

where $E = N_1 + \sum_{n>1} 2nN_n$ is a sum of harmonic oscillator energies, $\mathcal{H}_{N_1}$ is the Hermite polynomial of degree N_1, and $\mathcal{L}_{N_n}^{(n^2-3)/2}$ is the generalised Laguerre polynomial of degree N_n [10]. The gravitational part of the wave function, $\psi_E(a)$, must satisfy the equation

$$\left[-\frac{\partial^2}{\partial a^2} + a^2 - \lambda a^4 - 2E \right] \psi_E(a) = 0. \tag{2.5}$$

If we now imposed the usual wormhole boundary condition that requires the wave function to be damped when $a \to +\infty$, we would obtain $\psi_E = 0$. Actually, every non-vanishing solution of Eq. (2.5) is asymptotically oscillatory.

The potential term $(a^2 - \lambda a^4 - 2E)$ in Eq. (2.5) has two positive roots when $8E\lambda < 1$, namely, $a_{\pm} = (2\lambda)^{-1/2}[1 \pm (1 - 8E\lambda)^{1/2}]^{1/2}$. These turning points divide the sector of

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[1] ¿From now on, we will use a rescaled cosmological constant, $\lambda = \frac{\Lambda}{3}$, and set $\frac{2l_p^2}{3\pi} = 1$.

positive scale factors into three regions: two oscillatory, Lorentzian domains, $0 < a < a_-$ and $a > a_+$, and an exponential, Euclidean domain, $a_- < a < a_+$ (for $8E\lambda \geq 1$ we have an oscillatory behaviour for all $a > 0$). When $8E\lambda \ll 1$, we can perform a WKB analysis in the exponential domain, far from the turning points. This analysis reveals that there are two possible behaviours for the wave function in this region, namely, the leading term in the WKB approximation can either increase or decrease exponentially for increasing a. For $a_- < a < a_+$, the leading-order WKB approximation is

$$\psi_E(a) \simeq \sum_{\delta=\pm 1} A_\delta \, (a^2 - \lambda a^4 - 2E)^{-1/4} \exp\left(\delta \int_{a_-}^a da'(a^2 - \lambda a^4 - 2E)^{1/2}\right), \qquad (2.6)$$

where $A_\pm$ are constants.

When $8E\lambda \ll 1$ and $\lambda \ll 1$, this approximation is valid [12] at least for scale factors near the bottom of the potential, $a_m = 1/\sqrt{4\lambda}$. In particular, note that (for fixed E) the approximation is valid in a region that overlaps with that of large scale factors if λ is sufficiently small. Demanding that the leading-order WKB approximation is exponentially damped in the region of large scale factors well inside the horizon does not totally remove a subdominant contribution from the increasing exponential ($\delta = 1$) in Eq. (2.6). In the flat case ($\lambda = 0$), the increasing exponential would dominate the wave function when a becomes unrestrictedly large, even if A_+ is considerably small. Hence, the condition that $\psi_E(a)$ is exponentially damped for asymptotically large scale factors actually implies $A_+ = 0$ if the cosmological constant vanishes. In our case, however, the requirement of exponentially damped behaviour only implies that the quotient $|A_+|/|A_-|$ (which can depend on λ, E and Planck length) has to be small enough to suppress the contribution of the increasing exponential in $\psi_E(a)$ when the scale factor approaches the horizon size.

On the other hand, the de Sitter wave functions display an oscillatory behaviour for scale factors larger than the horizon size, $a > a_+ \simeq 1/\sqrt{\lambda}$, owing to the presence of the cosmological term in the potential. In this region, the WKB approximation gives

$$\psi_E(a) \simeq \sum_{\delta=\pm 1} A'_\delta \, (2E - a^2 + \lambda a^4)^{-1/4} \exp\left(\delta i \int_{a_-}^a da'(2E - a^2 + \lambda a^4)^{1/2}\right), \qquad (2.7)$$

which is always valid for sufficiently large scale factors. When $A'_+ = 0$, the gravitational wave function $\psi_E(a)$ is completely characterised in the region beyond the horizon by the fact that it represents a purely outgoing wave. Moreover, integrating that solution backwards in a, one obtains a unique wave function among those which exhibit an exponentially damped WKB behaviour in the Euclidean region.

Based on the above discussion, we now introduce the following proposal. For sufficiently small cosmological constant, one can interpret as quantum wormholes in de Sitter spacetime the wave functions (2.4) whose gravitational part: *i*) admits a WKB approximation, which is exponentially damped, in the interval of large scale factors well inside the Euclidean domain, and *ii*) corresponds to an outgoing mode in the region beyond the horizon. Any linear combination of such wave functions represents also a quantum wormhole state. Our proposal restricts the existence of wormhole wave functions to the sector of matter energies with $8E\lambda \ll 1$, since it is only then that an Euclidean region with the required properties exists. As we have seen, this proposal picks out a unique wave function for each value of E. Finally, notice that the condition that the wave function includes only outgoing gravitational modes for very large scale factors is similar in

spirit to the tunnelling proposal of Vilenkin [13], although in our case the tunnelling to the large Lorentzian region does not occur from "nothing", but from another Lorentzian domain, namely that with small scale factors.

A motivation for this proposal comes from the following considerations. For sufficiently small λ , let us choose a matter energy such that $8E\lambda \ll 1$. The turning point a_-, which corresponds to the wormhole throat, is then approximately equal to $\sqrt{2E}$, which is the throat size of an asymptotically flat wormhole with the same matter energy [11]. In the interval $(0, a_-)$ the de Sitter wormhole wave function has an oscillatory behaviour. This behaviour is similar to that displayed by a flat wormhole [3] and describes a Lorentzian closed Friedmann-Robertson-Walker spacetime, i.e. a baby universe. Furthermore, in the region $\sqrt{2E} \ll a \ll a_+ \simeq 1/\sqrt{\lambda}$, where the potential is dominated by the term a^2, one can parallel the line of reasoning discussed by Hawking and Page in the flat case to conclude that the main contribution to the wormhole wave function in the saddle point approximation must be given by the exponential of the surface term $-\frac{1}{2}\int \sqrt{h}|K|d^3x$, evaluated on the section of constant time with scale factor a. Here, K is the trace of the extrinsic curvature. It is then easy to see that one arrives precisely at the exponentially damped WKB behaviour that we have proposed for the wave function $\psi_E(a)$ in the region of scale factors under consideration. In order to select a wave function among those that possess this exponentially damped behaviour, one needs to generalise the arguments given by Hawking and Page to the region of scale factors beyond the horizon. In this region, the saddle points are Lorentzian and describe asymptotically de Sitter geometries that either expand or contract from a scale factor a larger than the horizon size. When the dominant saddle points are expanding (contracting) geometries, the wave function includes only outgoing (ingoing) gravitational modes beyond the horizon. In the flat case, the condition that the wormhole wave functions are asymptotically damped reflects the fact that the tunnelling occurs from the baby universe to the region of asymptotically large three-geometries, but not in the opposite direction. It then seems natural to generalise this condition to the de Sitter case by imposing that, beyond the cosmological horizon, there are only outgoing gravitational modes, so that the wave function describes in fact a tunnelling from small to asymptotically large scale factors. In this sense, our proposal provides a natural generalisation of the asymptotic wormhole boundary condition once it is assumed that the size of the cosmological horizon $(1/\sqrt{\lambda})$ is sufficiently large. The de Sitter wormhole wave functions selected by this proposal have a behaviour that is similar to that of the asymptotically flat wormholes both in the baby universe sector and in the region of large scale factors much smaller than the horizon size.

Since $E\lambda$ is the square of the rate between the wormhole throat (of order $\sqrt{E}$) and the horizon size of de Sitter space $(1/\sqrt{\lambda})$, the condition $8E\lambda \ll 1$, that we have used in our discussion, allows us to regain the picture of wormhole connections in a background spacetime. This picture would break down for quantum states with $8E\lambda \gtrsim 1$, because these states describe quantum fluctuations whose characteristic size is of the order of or greater than the cosmological horizon of the de Sitter background. Hence, we will not regard the states with $8E\lambda \gtrsim 1$ as quantum wormholes.

Finally, we will see in the next section that the wave functions (2.4) with the above choice for $\psi_E(a)$ can be interpreted as de Sitter wormholes with a definite number of particles in the Euclidean vacuum for de Sitter space [14, 15], that is, the vacuum which is conformally related to the natural vacuum for flat spacetimes [7].

3 Effective interactions.

Following a procedure developed by Hawking [2], and used in Ref. [7] for the anti-de Sitter case, we will now deduce the explicit form of the effective interactions produced by wormholes in de Sitter spacetime. In order to do this, we must first calculate the matrix elements of products of matter fields between a vacuum in de Sitter spacetime and an arbitrary wormhole state, $|\Psi_\alpha\rangle$,

$$\langle\Psi_\alpha|\phi(x_1)\phi(x_2)|0\rangle. \tag{3.1}$$

Here, we have considered only two matter fields for symplicity. In this section, we will choose the vacuum $|0\rangle$ to be the Euclidean vacuum. In Sec. 4, we will comment on the consequences of different choices of vacuum. Since our aim is to determine the effects of wormholes in scales greater that the wormhole scale, x_1 and x_2 represent points in regions of the Euclidean de Sitter spacetime far from the tiny wormhole end. For scale factors a in the Euclidean region, the state $\phi(x_1)\phi(x_2)|0\rangle$ is then given, in a (a,χ)-representation, by a Euclidean path integral over geometries and matter field configurations with initial values a and χ for the scale factor and the matter field, respectively, and compatible with the condition $E_\chi = E_a = 0$ at an arbitrary final time. Here, $E_\chi = \frac{1}{2}\sum_n(n^2\chi_n^2 - \pi_{\chi_n}^2)$ is the energy of the matter field, and $E_a = \frac{1}{2}(a^2 - \lambda a^4 - \pi_a^2)$ is the energy associated with the scale factor. Fixing the variables E_χ and E_a in this way at a final time τ_f selects the Euclidean vacuum for the matter field (the classical solutions for the scalar field with $E_\chi = 0$ correspond to the Euclidean mode decomposition of the field) and makes the action invariant under time reparametrizations that coincide with the identity at the initial time, but are arbitrary at τ_f [16]. Therefore, the particular value chosen for the final time becomes irrelevant.

An estimate of the path integral can be obtained by means of a saddle point approximation. As far as the low-energy regime is concerned, the geometrical saddle point can be taken to be pure de Sitter space outside a subtracted sphere of radius a that surrounds the wormhole insertion. Then, the saddle point solution for the conformal scalar field must satisfy the equation $(\Box - 2\lambda)\phi = 0$, where $\Box$ is the Laplacian for the de Sitter four-sphere. If $\phi(x')$ is a saddle point solution, $\phi_f(x')$, the transform of ϕ under an element f^{-1} of the group of isometries $SO(5)$, is also a solution. As a consequence, one has to average over the group $SO(5)$. Recalling that the four-sphere S^4 (i.e., Euclidean de Sitter spacetime) and the coset space $SO(5)/SO(4)$ are isomorphic, the integral over $SO(5)$ can be performed as follows

$$\int_{SO(5)} df\, F(f) = \int_{S^4} d^4x\sqrt{g(x)}\int_{SO(4)} dh\, F(xh), \tag{3.2}$$

h being a generic element of the isotropy group $SO(4)$ and g the determinant of the metric on the four-sphere. This integral can be interpreted as an average over the positions and orientations in which a wormhole end can be connected.

In Ref. [7] it was shown, by treating each mode separately, that the different saddle points can be expressed in terms of the propagator of the matter field as

$$\phi_{xh}^n(x') = \mathcal{M}_h^n \cdot \Theta_n G(x',x), \tag{3.3}$$

where G is the propagator associated with the Euclidean vacuum, and $\mathcal{M}_h^n$ is a constant tensor of range $n-1$ that is completely symmetric and vanishes under contractions of any pair of indices. This tensor contains all the dependence on the rotation group, as well as on a and χ_n (i.e., the values of the scale factor and the matter field coefficients on the sphere that has been subtracted to de Sitter space). The function $\Theta_n G$ is constructed by completely symmetrising a product of $n-1$ covariant derivatives acting on the x-dependence of the propagator, $\nabla^{\mu_1} \cdots \nabla^{\mu_{n-1}} G$, and subtracting all its traces [7]. Finally, the dot in (3.3) denotes the scalar product in the linear space of tensors with the explained symmetries. Taking into account that $\mathcal{M}_h^n = R(h)\mathcal{M}^n$, where $R(h)$ is the appropriate irreducible representation of the rotation group, which satisfies

$$\int_{SO(4)} dh R(h) \otimes R(h) = 1 \otimes 1 \tag{3.4}$$

(with $\otimes$ the tensor product), the average over orientations of the product of the two matter field solutions leads to

$$\int_{SO(4)} dh \phi_{xh}^n(x_1)\phi_{xh}^n(x_2) = (\mathcal{M}^n \cdot \mathcal{M}^n)\left(\Theta_n G(x_1, x) \cdot \Theta_n G(x_2, x)\right). \tag{3.5}$$

Hence, the complete expression for the quantum state $\phi(x_1)\phi(x_2)|0\rangle$ has the form

$$F(a, \chi_n) \int_{S^4} d^4 x \sqrt{g(x)}(\Theta_n G(x_1, x) \cdot \Theta_n G(x_2, x)). \tag{3.6}$$

In the function $F(a, \chi_n)$, the dependence on χ_n is of the form $(\chi_n)^2 e^{-\frac{1}{2}n^2(\chi_n)^2}$ [7], where the factor of $(\chi_n)^2$ comes from the product $(\mathcal{M}^n \cdot \mathcal{M}^n)$ and the exponential factor from the evaluation of the action on the classical solution.

The orthogonality of the Hermite and Laguerre polynomials that appear in (2.4) implies then that the state $\phi(x_1)\phi(x_2)|0\rangle$ has non-vanishing projections only on the vacuum and either on the $N_n = 1$ state for $n > 1$ or the $N_1 = 2$ state for the homogeneous case. We can therefore interpret $\Psi_{N_n=1}$ with $n > 1$ and $\Psi_{N_1=2}$ as quantum wormholes that, in the Euclidean vacuum, contain a two-particle rotationally invariant state and two homogeneous particles, respectively. A similar analysis can be applied as well to the three-point function and higher functions, extending the above interpretation to any wave function $\Psi_{N_1,\cdots,N_n\cdots}$ of the form (2.4) such that $(N_1 + \sum_{n>1} 2nN_n)8\lambda = 8E\lambda \ll 1$.

One can now deduce the expression of the interaction Lagrangian that, via the formula $\langle 0|\phi(x_1)\phi(x_2) \int d^4 x \sqrt{g(x)}\mathcal{L}_n^I(\phi(x))|0\rangle$, reproduces the matrix element (3.1) up to a constant factor. This Lagrangian must be of the form $\mathcal{L}_n^I = \Theta_n \phi \cdot \Theta_n \phi$, as can be seen by making use of Wick's theorem and noting that the operator Θ_n is linear [7]. For the lowest modes, for instance, the interaction Lagrangians turn out to be

$$\mathcal{L}_1^I = \phi^2, \qquad \mathcal{L}_2^I = \nabla^\mu \phi \nabla_\mu \phi, \qquad \mathcal{L}_3^I = \left(\nabla^\rho \nabla^\sigma \phi - \frac{1}{2}\lambda g^{\rho\sigma}\phi\right)\left(\nabla_\rho \nabla_\sigma \phi - \frac{1}{2}\lambda g_{\rho\sigma}\phi\right). \tag{3.7}$$

It is worth noting that, for $n \geq 3$, the form of the interactions differs from that obtained for flat space in terms that depend on λ, the square of the inverse horizon size. Therefore, the local interactions introduced by wormholes seem to depend on the large structure of spacetime. It then might happen that the constants of nature could be affected by contributions of cosmological origin owing to the existence of quantum wormholes.

4 Discussion and conclusions.

In this work, we have analysed the possible effects of the existence of wormholes in cosmological spacetimes with matter content. In this kind of spacetimes, it is necessary to generalise the standard, asymptotic wormhole boundary condition, because, when the spacetime possess cosmological horizons, no asymptotic Euclidean region exists. We have considered in detail the case of de Sitter spacetime, and extended the line of reasoning discussed by Hawking and Page for the case of flat wormholes. In this way, we have arrived at the following proposal. In a large de Sitter spacetime (i.e., when the cosmological constant is sufficiently small), it is possible to interpret as quantum wormhole states the wave functions that *i*) admit a WKB approximation with exponentially damped leading term in the region of large scale factors much smaller than the horizon size, and *ii*) contain only outgoing gravitational modes beyond the horizon. Unlike the situation found in the flat and anti-de Sitter cases, the existence of a cosmological horizon in de Sitter space poses an obstruction for the interpretation of a quantum state as a wormhole: the interpretation is feasible only in the sector of states with small matter energy. This restriction is necessary to guarantee that there exists a large Euclidean region in which the wave functions can have an exponentially damped behaviour. Without this restriction in the matter energy, the entire observable universe could be contained in the considered quantum fluctuation, so that it would be imposible to distinguish the interior of the wormhole from the background universe.

We have analysed the case of de Sitter wormholes with a conformal scalar field and discussed the effects of these quantum fluctuations in low-energy field theory. We have shown that the effective interactions produced by these wormholes differ from those induced by flat wormholes for $n = 3$ and higher harmonics. These differences are proportional to even powers of the inverse horizon size, i.e., to positive powers of the cosmological constant.

For other matter fields one would obtain similar results. As long as there exist quantum wormholes that admit the interpretation of small connections in a background spacetime, one can calculate their effective interactions in the following way. Given a field with spin s, each of the hyperspherical harmonics on the three-sphere in which we can decompose its true degrees of freedom carries an irreducible representation of the group $SU(2) \otimes SU(2)$, the universal covering of the isotropy group $SO(4)$. This irreducible representation is of type $(m/2 + s, m/2)$ or $(m/2, m/2 + s)$, where $m = n - s - 1$ is a non negative integer, n is the mode of the considered harmonic, and the lowest mode is given by $n = s + 1$. Each harmonic gives rise to a different interaction Lagrangian. The explicit form of these Lagrangians would be $\Theta_n \Phi \cdot \Theta_n \Phi$, with Φ representing a matter field of spin s. Finally, the operator Θ_n can be constructed with the help of the symmetries that the corresponding hyperspherical harmonic possesses when we write it in a Cartesian basis [7, 8], as it was proved in Ref. [7] for integer spins.

Let us illustrate this point with two examples. We will first consider the case of de Sitter wormholes with a minimally coupled massless scalar field ϕ. In the process of subtracting all the traces of the completely symmetric product of covariant derivatives that act on the propagator (as was explained above for the conformal scalar field), one must take into account that the approximate saddle point equation is now $g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = 0$, instead of the equation that applies in the conformally coupled case, $(g^{\mu\nu}\nabla_\mu\nabla_\nu - 2\lambda)\phi = 0$. This results in changing the interaction Lagrangians for the third ($n = 3$) and higher

modes with respect to those obtained for the conformal field. For instance, the Lagrangian for the mode $n = 3$ takes the form $\mathcal{L}_I^3 = (\nabla_\mu \nabla_\nu \phi)^2$. In contrast with the conformal field case, the interaction Lagrangians for the minimally coupled scalar field in flat and de Sitter spaces differ then just in that partial derivatives in flat space are replaced with covariant derivatives in de Sitter space. Let us discuss now the case of an electromagnetic field. In this case, Φ represents the four-potential A_μ. The interaction Lagrangian for the lowest mode $n = 2$ reads $(\Theta_2 \Phi)^2 = (\nabla_\mu A_\nu - \nabla_\nu A_\mu)^2 = (F_{\mu\nu})^2$ [7]. One can then recursively construct $\Theta_n \Phi$ for higher harmonics by symmetrising $\nabla \Theta_{n-1} \Phi$ and subtracting all its traces. This subtraction is easily performed by noticing that $\Box F_{\mu\nu} = 4\lambda F_{\mu\nu}$, an equation that follows from $\nabla^\mu F_{\mu\nu} = 0$. Finally, the tensor $(\Theta_n \Phi)_{\mu_1 \mu_2, \mu_3 \cdots \mu_n}$ must be antisymmetric in its first two indices, symmetric with respect to all other indices, and vanishing when contracted in any pair of indices or when taking a cyclic sum over μ_1, μ_2, and any other index. These are the symmetries corresponding to the n-th mode of the transverse vector harmonics on the three-sphere, which are the true degrees of freedom of the electromagnetic field [7]. On the other hand, when dealing with the electromagnetic field (as well as with other fields of higher spin), another physical quantity comes into play: the helicity. The helicity distinguishes the (p, q) and (q, p) irreducible representations. This can be done by introducing operators $\Theta_{n\pm}$ that are the self-dual and anti-self-dual parts of Θ_n. Nonetheless, the interaction Lagrangians for positive and negative helicities turn out to coincide, because the cross product $\Theta_n \Phi \cdot {}^*\Theta_n \Phi$ is a topological invariant (as can be cheked by direct calculation).

In general, we find that the wormhole effective interactions have contributions owing to the presence of a cosmological horizon. The explicit form of such contributions depends on the specific matter content.

To conclude, we will make some comments on our choice of vacuum in Eq. (3.1). Throughout our calculations, the state $|0\rangle$ has been chosen as the Euclidean vacuum in de Sitter space. In Ref. [7], it was shown that, for anti-de Sitter wormholes, the choice of a particular vacuum among the family of maximally symmetric vacua is a matter of convenience. Once a vacuum is chosen, one can always construct a Fock space of quantum wormholes labelled by the number of particles that they contain, as referred to that vacuum. In the de Sitter case, however, the existence of the horizon scale implies that the Euclidean vacuum plays an special role. The total number of particles associated with this vacuum that a de Sitter wormhole can contain is restricted by the condition $8N\lambda \ll 1$. A quantum state $\Psi_{\bar{N}}$ with a definite number of particles $\bar{N}$ in another vacuum has non-vanishing projections in states with $8N\lambda \geq 1$ regardless of the value of $\bar{N}$, so that it cannot be interpreted as a quantum wormhole. Therefore, the Euclidean vacuum turns out to be special in the sense that only observers associated with it can interpret certain wormhole states as containing a definite number of particles.

Acknowledgments

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